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Longitudinal analysis applied to the analysis of height increment of pine stands

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SUMMARY

One of the most important characteristics in the determination of dendrometrical properties of a stand is the annual increments in the height of the trees. On the basis of these increments, the natural phases of tree life are detected. In the case considered here, the data relate to height increments of the main trunk of 25-year-old Scots pine trees. Our research deals with the application of longitudinal data analysis. Usually this analysis is used when measurements are taken for the same treatments at different time points. Calculation provides an answer to the question of which annual height increments differ. Based on the research we can conclude that the data are from the final senile phase.

Key words: contrast, height increment, longitudinal data analysis, Scots pine (Pinus sylvestris)

1. Introduction

The increment in the height of trees is a very important dendrometrical characteristic. The magnitude of this increment is indispensable for using methods to determine the productivity of the stand; see Grochowski (1960), Gieruszyński (1961), Bruchwald (1971, 1973, 1999a). Each year annual shoots develop from terminal buds formed in the previous vegetation season. The length of the annual shoot, grown in the vegetation season and lignified, does not change, which makes it possible to determine its length at any given time. The length of the annual shoot is a variable trait. The actual annual increment in the height of the tree is unstable and depends on the weather conditions during growth (in the vegetation year) and on the weather conditions in the previous year, particularly between July and September. This is a very important period because buds are formed and reserve substances are accumulated. The reserve substances are used for growing the sprouts in the following year (see Assmann, 1968).

For describing the general regularity of the process of annual increment in the height of trees, we use the height increment curve shown in Figure 1; for details see Assmann, (1968), Bruchwald, (1999b, 2002).

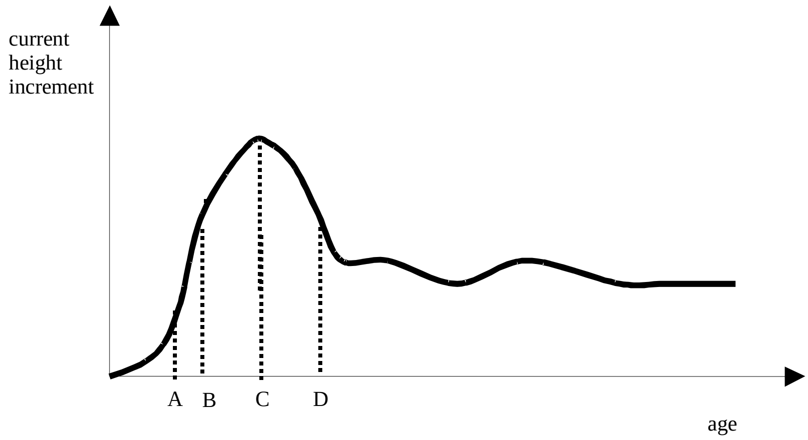

Figure 1. Increment in height of trees

The general regularity of the growing process is independent of the species of tree. Differences depend on the magnitude and length of natural phases. On the height increment curve we are able to fix the culmination point and the reversible points which determine typical phases of the correct height increment process. The first phase is determined by the convex segment from 0 to the first reversible point A. In these first years of a tree’s life – the youth (or juvenile) phase – the height increment is not large. According to some authors, this phase extends up to point B. Bruchwald (1999b, 2002) defined the juvenile phase as lasting until the time when the current increment (before culmination) is equal to the maximal value of the mean increment. The next period – a concave segment between A (or B) and D – is called the height season strength phase – the vigor phase. Trees have the largest annual height increments here, and during this time there appears culmination point C – the year with the greatest height increment. After a few years of similar height increment there comes the senile phase – a period of stabilized increments, much smaller than in the second phase, or sometimes at the same level (see, for example, Żółciak, 1963, Beker, 1998). The time of these periods depends mainly on the species of the tree and the climatic conditions; see Beker (1998).

The subject of our study is an application of longitudinal data analysis (sometimes called profile analysis: see the ISI International Glossary) to the analysis of height increments of trees based on the example of a pine stand.

2. Experimental material

The research was conducted on 150 sample trees coming from a 25-year-old pine stand on a sample plot area of 0.1 ha in the Zielonka Experimental Forest District. Sample trees were selected following the methodology developed by Draudt. Part of the data (in cm) is given in Table 1. The height increments were measured after the growing period. We studied trees aged between 19 and 25 years. For simplification, records of the increment in the period 19–20 years were indexed by "1", those in the period 20–21 years by "2", and so on.

Table 1. Annual increment of the main trunk for 25 trees in 6 years

Year / N^{o} of tree	1	2	…	6
1	48	25	…	14
2	45	60	…	35
…	…	…	…	…
25	68	77	…	55

In Figure 1, a spaghetti plot is used to visualize the time trends. The increments in height are plotted against year for each tree. These plots for visualizing the trajectories for all individual trees are called spaghetti plots or “individual plots”.

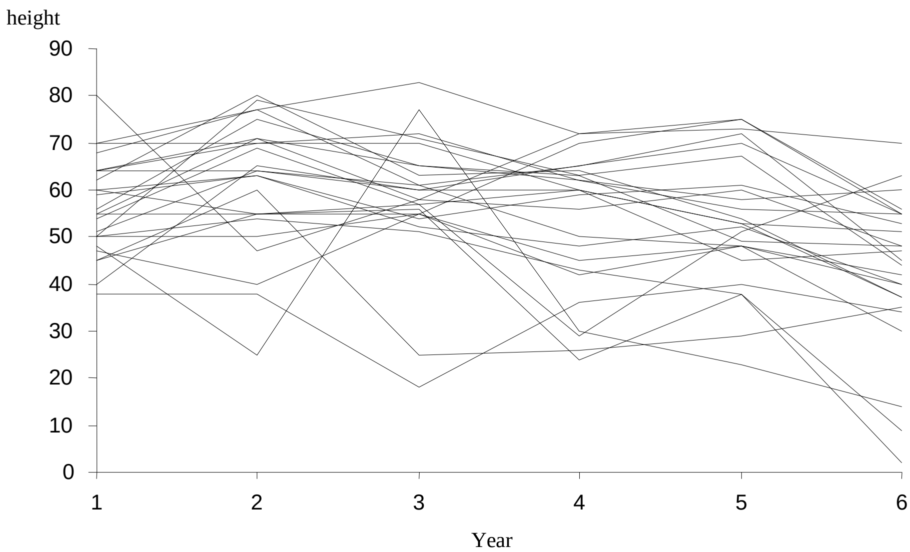

Figure 1. Annual increment of the main trunk for 25 trees in 6 years

In the present paper, we describe the application of longitudinal data analysis to determining properties of the trees. The size of the experimental material can be considered insufficient. In this paper the authors aim to present a method which is not often encountered in the literature relating to stand productivity analysis.

3. Model

In the paper, we consider the analysis of an experiment studying the behavior of experimental units – trees – subject to different time and space conditions.

We consider the annual height increments of the main trunk in a period of 6 years. In this experiment, the measurements are made each year using the same experimental units. In such experiments, there are two species of factors: the levels of the first factor are investigated on different experimental units, whereas the levels of the second factor are investigated on the same units. The first factor is connected with the units' classification, while the second is called a profile factor and is connected with the replication of measurements for a given profile of experimental units. Therefore, according to Morison (1990) the recorded results of measurements could be fitted into the linear model

$$y_{ij} = \mu + \mu_j + e_{ij}, \tag{1}$$

where y_{ij} is the annual increment of the height of the main trunk observed on the i th tree in the j th year, μ is the general mean, μ_j is the result of the j th year, e_{ij} is the random error, $i = 1,2,...,n,\ j = 1,2,..., p$. The variable includes the effect of interaction between the i th tree and j th year and experimental error. For a random vector of errors $\mathbf{e}_j = [e_{j1} \quad e_{j2} \quad \cdots \quad e_{jn}]'$, we have $\mathrm{E}(\mathbf{e}_j) = \mathbf{0}_p$ and $\mathrm{E}(\mathbf{e}_j \mathbf{e}'_{j'}) = \Sigma, \quad j \neq j',\ j, j' = 1,2,..., p,$ where $\mathbf{0}_p$ is the $p \times 1$ vector of zeros. This means that random vectors of errors for different experimental units (i.e. the trees) are independent. We are interested in testing the hypothesis that the mean increments of the height of trees during the 6 years are the same $\mathrm{H}_0 : \mu_1 = \mu_2 = \cdots = \mu_6$. The alternative hypothesis is that the mean height increments of the trees during the 6 years are not the same. The above hypothesis could be written in vector notation as:

$$\mathrm{H}_0 : \begin{bmatrix} \mu_2 - \mu_1 \\ \mu_2 - \mu_3 \\ \cdots \\ \mu_2 - \mu_6 \end{bmatrix} = \mathbf{0}_5 \qquad \mathrm{H}_1 : \begin{bmatrix} \mu_2 - \mu_1 \\ \mu_2 - \mu_3 \\ \cdots \\ \mu_2 - \mu_6 \end{bmatrix} \neq \mathbf{0}_5. \tag{2}$$

We test (2) using the Hotelling T^2 test:

$$T^2 = n\overline{\mathbf{y}}'\mathbf{C}'\left(\mathbf{CSC}'\right)^{-1}\mathbf{C}\overline{\mathbf{y}}, \tag{3}$$

where $\overline{\mathbf{y}} = [\overline{y}_1 \quad \overline{y}_2 \quad \cdots \quad \overline{y}_6]'$, $\mathbf{C}$ is the matrix of contrasts and $\mathbf{S}$ is the covariance matrix for the years. Assuming the null hypothesis to be true, the random variable T^2 given by (3) has T^2 distribution with $p-1$ and $n-p+1$ degrees of freedom. If $T^2 > T^2_{\alpha,p-1,n-p+1}$ then we reject the null hypothesis.

4. Results

In our account, we take p =6 and n =25. According to Morison (1990), the data should be normally distributed. Because the data are not normally distributed, we use the Box–Cox transformation (1964) $x_{ij}(\lambda) = \lambda^{-1}(y_{ij}{}^{\lambda} - 1)$ with the parameter $\lambda = 0.5$, where x_{ij} denotes the value after the Box–Cox transformation and y_{ij} the value before the transformation.

For analysis of the properties of the trees, we choose longitudinal data analysis, which permits deep analysis of the experiment. The estimates from the trial of transformed data are given as

$$\overline{\mathbf{x}} = \begin{bmatrix} \overline{x}_1 \\ \overline{x}_2 \\ \overline{x}_3 \\ \overline{x}_4 \\ \overline{x}_5 \\ \overline{x}_6 \end{bmatrix} = \begin{bmatrix} 12.93 \\ 13.57 \\ 13.15 \\ 12.41 \\ 12.49 \\ 10.71 \end{bmatrix}, \ \mathbf{S} = \begin{bmatrix} 1.84 & 1.07 & 1.41 & 1.39 & 0.95 & 0.90 \\ 1.07 & 3.63 & 1.28 & 2.15 & 1.78 & 2.12 \\ 1.41 & 1.28 & 4.16 & 2.16 & 1.43 & 0.96 \\ 3.39 & 2.15 & 2.16 & 4.78 & 3.56 & 4.50 \\ 0.95 & 1.78 & 1.43 & 3.56 & 3.91 & 4.25 \\ 0.90 & 2.12 & 0.96 & 4.50 & 14.25 & 9.94 \end{bmatrix} \tag{4}$$

To answer the question of whether the differences between annual mean increments of heights of the main trunk are the same, we test the hypothesis (2). Let us consider the contrast matrix

$$\mathbf{C} = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}. \tag{5}$$

For the matrix (5) the value from (3) is equal to 23.076, whereas $T^2_{0.05,5,20}$=18.268. Thus, we reject the hypothesis that the differences between annual mean increments of heights of the main trunk are the same. Thus we are interested in determining which annual increment differs from the others. In order to determine which annual increments are not the same we use the simultaneous confidence intervals of Roy and Bose (1953):

$$\mathbf{b}_k\overline{\mathbf{x}} - \sqrt{\frac{1}{n}\mathbf{b}_k\mathbf{S}\mathbf{b}_k^{'}}\; T_{\alpha,p-1,n-p+1} \leq \mathbf{b}_k\boldsymbol{\mu} \leq \mathbf{b}_k\overline{\mathbf{x}} + \sqrt{\frac{1}{n}\mathbf{b}_k\mathbf{S}\mathbf{b}_k^{'}}\; T_{\alpha,p-1,n-p+1},$$

where $\mathbf{b}_k$ is the k th row of the contrast matrix given in (5), where $\overline{\mathbf{x}}$ and $\mathbf{S}$ are given in (4), $T^2_{0.05,5,20}$=18.268, $k=1,2,...,5$. For particular k the confidence intervals are given as [-0.832, 2.112], [-1.425, 2.265], [-0.475, 2,792], [-0.529, 2.689], [0.396, 5.324]. If the k th confidence interval contains 0, then the difference between the second year and the k th is not significant. If the k th confidence interval does not contain 0, the difference between the second year and the k th is significant. Because the first four intervals contain 0, the differences between the second year and years 1, 3, 4 and 5 are not significant from the statistical point of view, whereas the last (5th) interval does not contain 0, and so the differences between the 2nd and the 6th year are significant.

5. Conclusions

Based on the experimental material consisting of 25 trees from Zielonka Experimental Forest District, we analyzed the height increment using longitudinal data analysis. The analysis gives an answer to the question of whether the height increments of the trees are equal over 6 years. Because there are some differences between the height increments of trees in the age range of 19–25 years, we can determine in which year the height increment was different than in the others. It is very important that, although we can determine the differences, we are not able to establish the specific trend of these changes. Although the trees are young, in terms of the height increments they are in the last – senile – phase. However these trees still have the ability to increase in thickness.

6. Discussion

The problem of determining the limits of these phases has been considered in the literature. For example, Żółciak (1963) has shown the culmination point appears between 15 and 20 years, while Tomusiak and Zarzyński (2001) place it between 11 and 25 years. According to research by Michalak (1970), carried out in the Augustowski Forest at different biosocial locations, the culmination point is the 17th year.

The present paper describes the application of longitudinal data analysis for the analysis of experimental material consisting of 25 trees from Zielonka Experimental Forest District. However, in many situations, the sample size available is not enough large for the analysis. Because of this, our paper serves to present the method rather than a complete analysis.

There are reports in the literature of longitudinal analysis for stands. For example, in Mehtätalo (2005) longitudinal analysis of the data was applied by estimating the models as random effects models using two nested levels: stand and measurement occasion. Moreover, Lukas and Diggle (1997) report on the statistical analysis of a longitudinal study of repeated measurements over time which was used to investigate relative differences in the growth of Sitka spruce and Norway spruce seedlings during summer exposure to ozone over three growing seasons. Moreover a paper by Lappi (1997) presents a simultaneous statistical analysis of height curves using longitudinal analysis.

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